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SGT Density with Conditional Volatility, Skewness and Kurtosis in the Estimation of VaR: A Case of the Stock Exchange of Thailand

Golf Ataboonwongse
Thammasat University

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Abstract

One of primary tools used to assess the financial risk is Value-at-Risk (VaR). It turns to be a standard measure of downward risk among financial intermediaries and regulators recently as it summarizes the risk into just a single and easy-to-understand number.

Despite the simplicity of VaR's concept, an accurate calculation of VaR is still statistically challenging. This research aims to propose an alternative approach which is believed to provide more accurate VaR rather than the traditional ones. Instead of the conventional Gaussian distribution, the more flexible skewed generalized t (SGT) density function is assumed for return series. Its volatility is characterized by eight types of generalized autoregressive conditional heteroscedasticity ($GARCH$) process. Meanwhile, the conditional skewness and kurtosis is modeled to exhibit time-varying feature by their past information set and autoregressive term. Daily returns on SET index will be used to explore the performance of estimated VaR. The finding shows that this new approach can provide more accurate and robust estimates of the actual VaR threshold, especially with $TS - GARCH$ model, than any other approaches that have been applied earlier.

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Chapter 1 Introduction

A primary tool used to assess the financial risk is Value-at-Risk (VaR). It is defined as the worst loss over a target horizon such that there is a low, pre-specified probability the actual loss will be larger[1]. The definition can be extended in term of statistics that VaR is the threshold value of the variable of interest when downside risk occurs given time horizon and coverage probability. For instance, when VaR of daily return on SET index is reported as −2.5 at 95% confidence level, it means that there is 5% of probability that the daily return on SET index would drop to −2.5. Its greatest advantage can be seen easily that it summarizes the risk in consideration into a single and easy-to-understand number. No doubt this explains why VaR is becoming an essential tool as a standard measure of downside risk in financial risk assessment among financial intermediaries and regulators.

The Bank of International Settlements (BIS) emphasizes this importance by introducing its application through the Basel II Accord, which is a new capital standard imposed on financial institutions such as commercial bank, security house, and insurance company[2]. While the precedent Basel I Accord provided the first step toward a safe and sound financial system by setting the minimum capital requirements of commercial banks to guard against credit risks, it was argued that the capital requirements are too simplistic and did not reflect the true risks of these financial institutions. The regulators amended this problem by establishing Basel II

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[1] The definition was proposed in Jorion (2007, ch.5).

[2] See Jorion (2007, ch.3).

Accord relied on risk-based capital charges that are better reflect the economic risks assumed. The concept of these new standards is generally based on the calculation of credit and market risks by VaR methods[3]. Hence, the accurate calculation of VaR is important to the minimum standard requirements of financial institutions and certainly the strength of soundness and stability of the international banking system as well.

Despite the simplicity of VaR's concept, an accurate calculation of conditional VaR[4] is still statistically challenging. It is classified broadly into two branches, empirical and parametric distribution approaches. This research focuses on the latter group, parametric VaR, which seems to produce more accurate measures of VaR given that the distributional assumption is realistic[5]. In order to compute the parametric VaR, the parameters of assumed distribution must be estimated first. Then, after the process of estimation and statistical inference of such parameters

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[3] See Jorion (2007, ch.3), Basel (2006) and Basel (2009). The Basel documents are available at www.bis.org.

[4] The term 'conditional VaR' is replaced intentionally to emphasize the use of $ARCH$ model proposed by Engle (1982) to compute VaR measure. The $ARCH$ process is used to model the volatility clustering which is one of stylized facts of financial time series by conditioning the variance of residual on the past information set. Although VaR can be computed without employing $ARCH$ process but simply assume normality of returns distribution, there is substantial empirical evidence confirming that the returns distribution is not characterized by normal distribution but by the 'stylized facts' of fat tails, high peakedness (excess kurtosis) and skewness. See chapter 2 for further details.

[5] See Jorion (2007, ch.5).

have been done, the corresponding parametric *VaR* will be calculated directly as a function of the estimated parameters. Thus, the main issue of accurate parametric *VaR* computation is whether the distributional assumption is realistic since dependable estimation and statistical inference can be performed accordingly.

The general approach used in Basel II Accord for parametric *VaR* computation assumes that the distribution of returns is normal distribution. Then, the conditional distribution of returns standardized by subtracting its mean and dividing its standard deviation will be transformed into standard normal distribution. As a consequence, the computation of conditional *VaR* is simplified considerably. The problem of finding conditional *VaR* at a given horizon and confidence level c is equivalent to finding the deviation such that the area to the left of standard normal distribution of standardized return is equal to $1 - c$.[6] Notwithstanding, there is substantial empirical evidence that the conditional distribution of standardized returns are not standard normal distribution and have fatter tail than are compatible with the standard normal distribution and also have volatility clustering.[7] It refers that the conditional *VaR* calculation approach rooted from standard normal distributional assumption in Basel II Accord will conduce towards less *VaR* measures than actual. These inappropriate computations of *VaR* lead to an inadequacy of minimum capital requirement of financial institutions and therefore, the instability of international financial system. A new method of *VaR* computation must be sought in order to provide greater descriptive validity of financial risk measures.

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[6] Then it can be transformed back from standardized returns to the real value of returns by multiplying its standard deviation and adding its mean.

[7] See Mandelbrot (1963a, 1963b) and recent book by Mills and Markellos (2008, ch.7) and references contained therein.

This research builds on Bali et al. (2008), who proposed a new method of generalized $ARCH$ ($GARCH$) models to compute VaR. In place of traditional standard normal distribution assumption, the standardized residuals are assumed to follow the skewed generalized t (SGT) distribution[8] whose conditional variance, skewness and kurtosis parameters follow autoregressive process as in $ARCH$ model. The finding from this method by using both daily returns data on the Center for Research in Security Price (CRSP) equal-weighted and value-weighted S&P500 indices indicates similar results that the conditional $SGT - VaR$ model provides very accurate and robust estimates of the actual VaR thresholds and better than any other approaches that have been applied to compute conditional VaR in earlier studies.

As a member of the world financial society, Thailand has implemented the Basel II Accord since 2008. Due to the fact that most of regulations applied to financial institutions are based on Basel II Accord, the validity of method for computing precise VaR is extremely important to the regulations on financial institutions. Nevertheless, there is no research investigating the alternative method of VaR computation in case of Thailand. This research is initiated with the hope that it will be beneficial to improve the regulations in financial market imposed by the Bank of Thailand; the Securities and Exchange Commission; and the Stock Exchange of Thailand. Furthermore, we look forward that this improvement in financial regulations will strengthen soundness and stability of Thailand's financial system and also prevent the financial crisis that may occur in the future at the end.

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[8] See Theodossiou (1998).

Research Objective:

To investigate the performance of *SGT* density with *GARCH* model of conditional volatility, skewness and kurtosis in terms of power to predict the *VaR* threshold in the case of the Stock Exchange of Thailand.

Expected outcomes

We expect that the new approach proposed in this research will provide more accurate estimates of actual *VaR* threshold than the traditional one.

Chapter 2 Literature Review

Since the conditional parametric VaR can be computed straightforwardly after the distribution of returns has been estimated, the main issue to consider in the computation of VaR is whether the distributional assumption of underlying returns is realistic. The standard assumption since 1960s has stated that financial prices follow geometric Brownian motions and, thus, logarithmic returns are a normal distribution. This assumption is justifiable from aspect of both finance and econometrics. A variety of financial models including primary and derivative asset pricing, portfolio optimization and risk management are built upon the normality assumption, which states that expected returns and risks in a multivariate normal financial world can be completely described probabilistically by using just means, variances and covariances. In a practical viewpoint, normality assumption offers tractability and simplicity in computation. Furthermore, the assumption is supported theoretically in econometrics by central limit theorem (CLT), which states that the sum of identical and independent ($i.i.d.$)[9] random variables with finite mean and variance will asymptotically converge to a normal distribution. Due to the advantages of assuming normality and the fact that independence was considered to hold reasonably well for various financial return series – e.g. see Cootner (1964) and Fama (1970), there is no doubt that the normal distribution quickly became a standard assumption in finance

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[9] It is not necessary or sufficient condition for theoretically consistent financial prices, however: see, for example, LeRoy (1973), Lucas (1978), Frankel and Froot (1988) and Goldberg and Frydman (1996).

and, therefore, the standard approach on the computation of VaR is based on normality assumption.

Notwithstanding, there are many empirical research on returns distributions since 1960s suggesting that the distributions are not characterized by normality but by the 'stylized facts' of fat tails, high peakedness (excess kurtosis) and skewness: see, for example, Kon (1984), Badrinath and Chatterjee (1988) and Miittnik and Rachev (1993a) and Rachev, Menn and Fabozzi (2005). The above findings implied that the VaR measurement computed from normality assumption would produce inaccurate VaR, which is less than the actual. Accordingly, researchers emphasized these stylized facts by assuming other distributions that have fat tails and high peakedness.

On top of this development, Mandelbrot (1963a, 1963b) proposed using the stable distribution[10], which includes the normal distribution as a special case, to model the fat-tailed nature of standardized returns. Stable distributions allow fat tails by displaying a power-declining tail rather than an exponential decline, as in the case with the normal[11]. Since then, the stable distributions have been found that provides a good fit to a wide variety of returns: see, for example, Ghose and Kroner

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[10] It is also known as the stable Paretian, Pareto-Levy or Levy flight distributions.

[11] Roughly speaking, if the characteristic exponent of characteristic function of stable distribution is two, the distribution will be normal and all moments are finite, whereas if the characteristic exponent is less than two, all moments greater than exponent value are infinite. In the latter case, it allows the possibility of infinite variance of the stable distribution, which is the cause of the fat tails. For more details, see Mills and Markellos (2008, ch.7) and Feller (1966).

(1995). Out of its empirical usefulness, the stable distributions are also theoretically justifiable for being an appropriate generating process for financial data. Mandelbrot (1963b) argues that such a justification arises from a generalization of the *CLT*, which states that if the limiting distribution of an appropriately scaled sum of *i.i.d.* random variables exists then it must be a member of the stable class of distributions, although these random variables have infinite variance[12]. Furthermore, the stable distribution has an important property known as stability or invariance under addition property. It implies that if daily returns, say, follow a stable distributions then, weekly, monthly, quarterly and annually returns, which can be viewed as the sum of the daily returns, will follow stable distribution as well[13].

In spite of many desirable properties of stable distributions, results considering their empirical appropriateness for describing financial returns have been conflicting. Generally, any supporting evidence seems to fade away as the sampling interval of returns increases. The estimates of variance seem to converge rather than being infinite. Nonetheless, the reluctance to replace the normal distribution by the stable family has been based only on the basis of practical convenience but not on any empirical or theoretical criteria. Some examples of these are that the stable distributions bring about severe mathematical problems – e.g. they have no simple analytical representation, parameters of distribution are notoriously

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[12] See the proof in Feller (1966), which is generalization of the moment requirements of the *CLT* and therefore expands the set of limiting distributions.

[13] For more detailed technical discussion of stable distribution, see Mandelbrot (1963a, Mandelbrot 1963b), Feller (1966), Brockwell and Davis (1996), Mittnik and Rachev (1993a, 1993b), Samorodnitsky and Taqqu (1994) and Rachev, Menn and Fabozzi (2005).

difficult to estimate, standard asymptotic theory is inapplicable, etc. In addition, the direct applications within standard finance theory framework, which requires finite second and often higher moments, are prohibited by the infinite variance property of the stable distributions.

Alternative approach to model the returns distributions has been first introduced by Engle (1982) and is known as the autoregressive conditional heteroscedasticity, or $ARCH$, process[14]. It embed a formal stochastic model of volatility in the series of returns itself in order to represent the stylized fact of volatility clustering. A simple way is to allow the conditional variance of the process generating another time series. Even though a stationary process must have a constant variance, certain conditional variances can change over time either certain discrete points in time or continuously. Engle (1982) imposed autoregressive, or AR, process on the square of disturbances. To obtain more flexibility, Bollerslev (1986) proposed the generalized $ARCH$ ($GARCH$) process, which is considered as imposing autoregressive moving average, or $ARMA$, process on the square of disturbances. Let a regression model with $GARCH$ disturbances be

$$y_t = \mathbb{x}_t'\beta + u_t. \tag{2.1}$$

Here $\mathbb{x}_t$ denotes a vector of predetermined explanatory variables, which could include lagged values of y_t. Suppose that

$$u_t = \sqrt{h_t} \cdot v_t, \tag{2.2}$$

where $\{v_t\}$ is an $i.i.d.$ sequence with zero mean and unit variance:

$$E(v_t) = 0 \qquad E(v_t^2) = 1, \tag{2.3}$$

and h_t evolves according to

$$h_t = \kappa + \delta_1 h_{t-1} + \delta_2 h_{t-2} + \cdots + \delta_r h_{t-r}$$
$$+ \alpha_1 u_{t-1}^2 + \alpha_2 u_{t-2}^2 + \cdots + \alpha_m u_{t-m}^2. \tag{2.4}$$

Expression (2.4) is the $GARCH$ model, denoted $u_t \sim GARCH(r,m)$. If u_t is described by

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[14] See Hamilton (1994, ch.21) and Mills and Markellos (2008, ch.5).

a $GARCH(r,m)$ process, then u_t^2 follows an $ARMA(p,r)$, where $p \equiv max\{m,r\}$[15]. It is convenient to condition on the first p observations ($t = -p + 1, -p + 2, ..., 0$) and to use observations $t = 1,2, ..., T$ for estimation. Let $\aleph_t = (y_t, y_{t-1}, ..., y_1, y_0, ..., y_{-p+1}, \mathbb{x}_t', \mathbb{x}_{t-1}', ..., \mathbb{x}_1', \mathbb{x}_0', ..., \mathbb{x}_{-p+1}')'$, denoted the vector of observations obtained through date t.

In order to estimate the parameters of a regression model with $GARCH$ disturbances u_t, Maximum Likelihood Estimation (MLE) is applied by assuming the conditional distribution of v_t, finding the sample likelihood function conditional on the past information sets $\mathbb{x}_t, \aleph_{t-1}$ and then the estimated parameters is the parameter value that maximize analytically or numerically the underlying sample likelihood function[16].

Traditional approach of the formulation of likelihood function for $GARCH$ process assumed that the conditional disturbances v_t have a conditional normal distribution. However, the unconditional distribution of disturbances u_t is non-normal distribution with heavier tails than a normal distribution, even if the conditional distribution is normal[17]. That is, instead of assuming fat-tailed distribution, i.e. stable class of distributions, to model stylized fact of returns; the approach of $GARCH$ process exhibits the same nature of returns distribution without the problem faced by the former. Since then, this alternative approach has turned to be an extremely popular in modelling returns distribution[18] and thus, be applied to compute conditional parametric VaR.

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[15] See Hamilton (1994, ch.21) for more details.

[16] See Greene (2008) ch.16.

[17] See Milhoj (1985) and Bollerslev (1986).

[18] See Mills and Markellos (2008).

With the best of our knowledge, the modifications of *GARCH* process can be classified into three categories. The first category exploits the *GARCH* model with alternative distributional assumptions. The conventional approach assumed that the distribution of conditional disturbances v_t is conditionally Gaussian distribution. However, this is not necessary. Examples of these attempts include the standardized t distribution by Bollerslev (1987), the normal-Poisson mixture distribution by Jorion (1988), the power exponential distribution by Baillie and Bollerslev (1989), the normal-log-normal mixture by Hsieh (1989), and the generalized exponential distribution by Nelson (1991). Many researchers have endeavored to examine their performance in many contexts. However, there is no general agreement among these distributional assumptions. For example, Wilhelmsson (2006) investigated the predictive ability of *GARCH*(1,1) model with various error distributions on S&P-500 index future returns. He found that the forecast can be improved by allowing for a leptokurtic error distribution. Chuang et al. (2007) compared the forecasting performance of linear *GARCH* model with different distributional assumptions in context of equity and foreign exchange market. It showed that a distribution does not always outperform the simple one.

Another category of interest is concerned with the modification of estimation procedure. Instead of the maximum likelihood estimation, some other estimation methods have been proposed. For example, Engle and Gonzalez-Rivera (1991) estimated semi-parametrically the density of disturbances, while Linton (1993) estimated adaptively the parameters of *ARCH* models in the presence of non-normal disturbances.

Finally, the last category on *GARCH* modification is devoted to model specification in the series of conditional variance h_t. These modifications lead to general class of *GARCH* models that is closely related to the study on asymptotic

properties, the existence of moments and other time series characteristics. Some of these are the *AGARCH* model of Engle (1990), the *IGARCH* model of Engle and Bollerslev (1986), the *EGARCH* model of Nelson (1991), the *GJR − GARCH* model of Glosten et al. (1993), *QGARCH* model of Sentana (1990), *TGARCH* of Zakoian (1994), *TS − GARCH* of Taylor (1986) and Schwert (1989c) and *NGARCH* of Engle and Ng (1993). Analogous to the modification in distributional assumption, there are many efforts investigating their performance. Yet, they ended up without any conclusions. For example, while Bralisford and Faff (1996) and Taylor (2004) found some evidences in favor of the *GJR − GARCH* model, Heynan and Kat (1994), Chong et al. (1999), London et al. (2000) came to the conclusion that *EGARCH* achieves the best predicting performance among others.

By and large, along with these three modifications, much research has focused mostly on the distributional assumption of v_t and the specification of conditional variance h_t since the maximum likelihood estimation possesses many desirable properties. As a consequence, Liu and Hung (2010) explored the relative importance of the distributional assumption and the asymmetric specification in terms of forecasting performance using S&P-100 stock index between 2001-2003. The authors divided a variety of models into two groups: symmetric *GARCH* models with various distributional assumptions, and various *GARCH* specifications with normal distribution. They found that modelling the asymmetric component is relatively more important than specifying the error distribution for improving volatility forecasts of financial returns in the presence of skewness, fat-tail, and leptokurtosis. Moreover, if asymmetric properties of *GARCH* specification are neglected, the *GARCH* model with normal distribution is preferable to those models with more sophisticated error distributions (Liu and Hung (2010)). However, the performance of the mixture models between asymmetric *GARCH* specifications and

the error distributions in the presence of skewness and excess-kurtosis has not been explored clearly yet.

Among a variety of modifications on *GARCH* process, Hansen (1994) argued that the standard assumption of *i.i.d.* v_t is not appropriate[19]. It can be shown by rewriting expression (2.1) as

$$y_t = \mathbb{x}_t'\beta + u_t = \mu_t + \sigma_t v_t, \qquad (2.5)$$

where μ_t and σ_t are, respectively, the mean and standard deviation of return y_t, conditional on $\mathbb{x}_t$ and $\aleph_{t-1}$. These conditional mean μ_t, and conditional standard deviation σ_t can be used to define the normalized error

$$v_t = \frac{y_t - \mu_t}{\sigma_t}, \qquad (2.6)$$

In most earlier regression model, the conditional distribution of normalized error v_t is simply assumed to be independent of the conditioning variable $\mathbb{x}_t$ and $\aleph_{t-1}$. Hansen (1994) argued that there is definitely no reason to expect the conditional distribution of derived variable v_t to be independent of the conditioning information. On the other word, there is no reason to assume that the only features of the conditional distribution v_t, which depend upon the conditioning information, are the mean and variance. In fact, it seems sensible that other features of distribution (for example; skewness and kurtosis) will depend on the conditioning information as well[20].

The reason why higher-order features of the conditional distribution have been ignored by the most applications may be because only the conditional mean and variance generate significant excitement. However, this lack of excitement does not imply that higher-order features should be completely ignored (Hansen (1994)). First, as the property of *MLE*, the efficient estimation requires a complete description of the conditional distribution that may include higher-order features. Second, it has

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[19] See also, Harvey and Siddique (1999), Jondeau and Rockinger (2003), Bali and Weinbaum (2007) and Brooks et al. (2005).

[20] See Gallant, Hsieh and Tauchen (1991) for example.

been shown in Baillie and Bollerslev (1992) that the accuracy of predictive distributions depends critically upon knowledge of the correct conditional distribution for the normalized error. Since the main purpose of conditional models is usually prediction, the complete description of conditional distribution should not be neglected. Third, in the context of asset pricing, where the price is determined by not just the conditional mean and variance but more complicated functions of the conditional distribution, the empirical models are incomplete unless the full conditional distribution is specified.

To achieve the goal in allowing the conditional density of v_t to depend on conditioning variable x_t and $\aleph_{t-1}$, Gallant, Hsieh and Tauchen (1991) proposed for using a series expansion about the Gaussian density to model the joint density of y_t and x_t. This innovative approach has the potential to disclose a lot of information regarding underlying distribution without having to impose a great deal of a priori information or structure. Notwithstanding, this approach has many drawbacks. First, the parameterization is not parsimonious and thus, requires very large data sets in order to fulfill a reasonable degree of precision. Second, the methods are computationally expensive and may be beyond many routine applications. Third, the consequence of the techniques may depend profoundly on choices of the number of expansion orders.

An alternative parametric approach to modelling the conditional density of the normalized error is initiated by Hansen (1994). This approach can be considered as a direct extension of Engel (1982)'s pioneering idea to model the conditional variance as a function of lagged errors. His suggestion is to select a distribution, which depends upon a lower dimension parameter vector and then, allow this parameter vector to vary as a function of the conditional variables, in the same way as in *ARCH* process of Engel (1982). Generally, any distributions with closed-form

density function are optional. Hansen applied the student's t distribution and a more general, the skewed student's t distribution to this approach. The findings are as expected that the shape parameters or higher-order parameters; for example, skewness and kurtosis parameters, of the conditional densities are found to be statistically significant. That is, higher-order features of conditional density do matter and therefore, the normalized variable v_t should not be assumed to be independent of the conditioning information x_t and $\aleph_{t-1}$.

Subsequently, Jondeau and Rockinger (2003) extended Hansen's (1994) idea by choosing the generalized student's t distribution introduced by McDonald and Newey (1988) as a density function of normalized error v_t. Jondeau and Rockinger focused their attention to various possible specifications for the dynamics of higher-order parameters. They indicated the spurious correlation problem[21] that will be encountered with such specifications and then provided diagnostic test to detect and solve the problem. They found that skewness and kurtosis parameters are persistent for many series.

The approach initiated by Hansen (1994) and extended by Jondeau and Rockinger (2003) is applied to estimate conditional VaR[22] in Bali et al. (2008). They argued that although earlier studies have tried to estimate the conditional mean and volatility of asset returns using the symmetric and asymmetric fat-tailed distributions; for example, skewed student's t distribution in Hansen (1994) and

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[21] It is also known as spurious persistence problem. See Jondeau and Rockinger (2003).

[22] The term "conditional VaR" refers to the VaR obtained from the approach initiated in Hansen (1994), which higher-order moment parameters of normalized error distribution are modeled to be a function of conditional variable.

generalized student's t distribution in Jondeau and Rockinger (2003), they did not assess the performance of alternative distributions in terms of their power to predict accurate conditional VaR thresholds. The skewed generalized student's t (SGT) distribution of Theodossiou (1998) has been introduced in Bali et al. (2008). It provides a flexible tool for modelling the empirical return distributions, which exhibit skewness, leptokurtosis and fat-tails. Also, the SGT nests many well-known highly flexible density functions; for example, skewed t distribution of Hansen (1994), generalized t distribution of McDonald and Newey (1988) and symmetric t distribution. The findings found the significance in time-varying higher-order parameters and also strongly indicated that the use of SGT distribution with time-varying parameters provide accurate predictions of catastrophic market risks and capture the rate of occurrence and the extent of extreme events surprisingly well in US stock market.

In case of Thailand, there is not much research on VaR. Lindo Jr. (2008), Sangiam (1997) and Komsan Piyamalmas (2002) applied the method of VaR to evaluate the risk associated with their interest. They assumed that the distribution of the underlying variable based on standard Gaussian or normal density function. In place of normality assumption, Vatcharachai (2006) goes further by assuming student's t distribution as an alternative in computing VaR of portfolio returns. The finding found that the VaRs computed from normal distribution is significantly higher than student's t distribution's at 97-99 percent confidence level. That is, if the exact distribution of portfolio returns is student's t distribution, the VaR computation based on standard normality assumption may mislead investors as it provides overestimated VaR and result in the loss of investors as they overlook the exact risk of the their portfolio.

Unlike the most research of *VaR* that puts their interest to continuous variable; i.e. returns, revenues, capital; the underlying variable in Yupaporn (2006) is discrete. She investigated the risk related to the bank loans to the clients, in which the variable of interest is the binary choices that the clients 'do' or 'do not' pay back the bank loans. Since the continuous distributional assumption of normality is not appropriate now, Yupaporn (2006) assumed the distribution of underlying variable to be Poisson-Binomial density function, which is a class of discrete distribution function.

Overall, the research of *VaR* computation in case of Thailand relies only on constant volatility over time. Hence, any *GARCH* processes are not included into the model to represent the stylized facts; for instance, fat tails, high peakedness and volatility clustering; and not even further the higher-order features of conditional distribution are considered in the computation of *VaR*.

Chapter 3 Methodology

3.1 The Model

To compute the precise conditional VaR, this literature builds on Bali et al. (2008). Firstly, the return distribution is assumed to follow SGT distribution, which generalized and nested conventional normal distribution and many flexible distributions in earlier research. Secondly, the conditional mean and variance parameters are modeled by employing eight variation of $GARCH(1,1)$ processes in order to represent the 'stylized facts' of fat tails and volatility clustering of returns distribution. Thirdly, since there is substantial empirical evidence[23] that the high-order moment parameters of the conditional density for standardized returns are significantly time-varying (Bali et al. (2008)), the conditional high-order moment parameters of the SGT distribution are computed through the approach initiated by Hansen (1994) and extended by Jondeau and Rockinger (2003). Finally, after all the time-varying parameters are estimated and whole information of the conditional return density is obtained, the conditional VaR is computed directly by the parametric distribution approach.

The aforementioned models are defined as follows:

$$r_t = \alpha_0 + \alpha_1 r_{t-1} + u_t = \mu_t + \sigma_t \cdot z_t, \tag{3.1}$$

$$g(\sigma_t) = \sigma_t, \sigma_t^2 \text{ or } ln(\sigma_t), \tag{3.2}$$

$$g(\sigma_t) = h(\sigma_{t-1}, z_{t-1}|\beta_0, \beta_1, \gamma) + \beta_2 \cdot g(\sigma_{t-1}), \tag{3.3}$$

where r_t is returns at time t; μ_t and σ_t are, respectively, the conditional mean and

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[23] They were mentioned in Chapter 2 already. See Hansen (1994), Jondeau and Rockinger (2003), and Bali et al. (2008) for further details.

conditional standard deviation of returns r_t based on past information set up Ω_{t-1} to time $t-1$; $u_t = \sigma_t \cdot z_t$ is the returns innovation at time t; $z_t = (r_t - \mu_t)/\sigma_t$ is standardized returns, which its density function is as follows:

$$f_Z(z_t|\lambda_t, \eta_t, \kappa_t) = C \cdot \left[1 + \frac{|z_t + \delta|^{\kappa_t}}{\left(\frac{\eta_t - 2}{\kappa_t}\right)(1 + sign(z_t + \delta) \cdot \lambda_t)^{\kappa_t} \varphi^{\kappa_t}} \right]^{\left(-\frac{\eta_t + 1}{\kappa_t}\right)},$$

(3.4)

where

$$C = 0.5\kappa_t \cdot \left(\frac{\eta_t - 2}{\kappa_t}\right)^{-\frac{1}{\kappa_t}} \cdot B\left(\frac{\eta_t}{\kappa_t}, \frac{1}{\kappa_t}\right)^{-1} \cdot \varphi^{-1},$$

(3.5)

$$\varphi = \frac{1}{\sqrt{\nu - \rho^2}},$$

(3.6)

$$\rho = 2\lambda_t \cdot B\left(\frac{\eta_t}{\kappa_t}, \frac{1}{\kappa_t}\right)^{-1} \cdot \left(\frac{\eta_t - 2}{\kappa_t}\right)^{\frac{1}{\kappa_t}} \cdot B\left(\frac{\eta_t - 1}{\kappa_t}, \frac{2}{\kappa_t}\right),$$

(3.7)

$$\nu = (1 + 3\lambda_t^2) \cdot B\left(\frac{\eta_t}{\kappa_t}, \frac{1}{\kappa_t}\right)^{-1} \cdot \left(\frac{\eta_t - 2}{\kappa_t}\right)^{\frac{2}{\kappa_t}} \cdot B\left(\frac{\eta_t - 2}{\kappa_t}, \frac{3}{\kappa_t}\right),$$

(3.8)

$$\delta = \rho\varphi.$$

(3.9)

The conditional standard deviation σ_t is assumed to follow various $GARCH(1,1)$-type models through the functional form of $g(\sigma_t)$ as in expression (3.2) and (3.3). The conditional volatility equations $g(\sigma_t)$ for eight variations of $GARCH(1,1)$[24] models are as follows:

(1) *GARCH model*

$$\sigma_t^2 = \beta_0 + \beta_1 \sigma_{t-1}^2 z_{t-1}^2 + \beta_2 \sigma_{t-1}^2,$$

(3.10)

(2) IGARCH: Integrated GARCH model

$$\sigma_t^2 = \beta_0 + (1 - \beta_2)\sigma_{t-1}^2 z_{t-1}^2 + \beta_2 \sigma_{t-1}^2,$$

(3.11)

(3) EGARCH: Exponential GARCH model

$$\ln(\sigma_t^2) = \beta_0 + \beta_1[|z_{t-1}| - E|z_{t-1}|] + \gamma z_{t-1} + \beta_2 \ln(\sigma_{t-1}^2),$$

(3.12)

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[24] By simple substitution, it reveals that the *GARCH* model is an infinite order *ARCH* model with exponential decaying weights for larger lags (See Engle and Bollerslev (1986)). Empirically, the *GARCH*(1,1) model is preferable in most cases (see the survey by Bollerslev et al. (1992)).

(4) GJR − GARCH: GARCH model of Glosten, Jagannathan and Runkle

$$\sigma_t^2 = \beta_0 + \beta_1 \sigma_{t-1}^2 z_{t-1}^2 + \gamma S_{t-1}^- \sigma_{t-1}^2 z_{t-1}^2 + \beta_2 \sigma_{t-1}^2, \tag{3.13}$$

where $S_{t-1}^- = 1$ for $\sigma_{t-1} z_{t-1} < 0$ and $S_{t-1}^- = 0$ otherwise.

(5) QGARCH: Quadratic GARCH model

$$\sigma_t^2 = \beta_0 + \beta_1 \sigma_{t-1}^2 z_{t-1}^2 + \gamma \sigma_{t-1} z_{t-1} + \beta_2 \sigma_{t-1}^2. \tag{3.14}$$

(6) TGARCH: Threshold GARCH model of Zakoian

$$\sigma_t = \beta_0 + \beta_1 \sigma_{t-1} |z_{t-1}| + \gamma S_{t-1}^- \sigma_{t-1} z_{t-1} + \beta_2 \sigma_{t-1}, \tag{3.15}$$

where $S_{t-1}^- = 1$ for $\sigma_{t-1} z_{t-1} < 0$ and $S_{t-1}^- = 0$ otherwise.

(7) TS − GARCH: GARCH model of Taylor and Schwert

$$\sigma_t = \beta_0 + \beta_1 \sigma_{t-1} |z_{t-1}| + \beta_2 \sigma_{t-1}, \tag{3.16}$$

(8) APGARCH: Asymmetric Power GARCH model

$$\sigma_t^2 = \beta_0 + \beta_1 [sign(z_{t-1}) - \gamma]^2 \sigma_{t-1}^2 z_{t-1}^2 + \beta_2 \sigma_{t-1}^2, \tag{3.17}$$

Note that the parameter γ represents an asymmetric volatility response to past positive and negative information shocks. The conditional volatility parameters $\beta_0, \beta_1, \beta_2, \gamma$ must satisfy the positivity and stationary constraints in each $GARCH(1,1)$ model in order to yield positive unconditional variance and stationary process. The conditional high-order moment parameters of the SGT density $\lambda_t, \eta_t, \kappa_t$ are modeled as follows[25]:

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[25] To avoid spurious correlation of $\tilde{\lambda}_t$, $\tilde{\eta}_t$ and $\tilde{\kappa}_t$, and their autoregressive terms of $\tilde{\lambda}_{t-1}$, $\tilde{\eta}_{t-1}$ and $\tilde{\kappa}_{t-1}$, which is mentioned in Jondeau and Rockinger (2003), we first estimate

$$\tilde{\lambda}_t = \lambda_0 + \lambda_1 z_{t-1},$$

$$\tilde{\eta}_t = \eta_0 + \eta_1 z_{t-1},$$

$$\tilde{\kappa}_t = \kappa_0 + \kappa_1 z_{t-1},$$

as intermediate step and verify that past observations affect $\tilde{\lambda}_t$, $\tilde{\eta}_t$ and $\tilde{\kappa}_t$. If the coefficient estimates of the lagged return innovation λ_1, η_1 and κ_1 are statistically significant, it indicates that the significant estimates of the autoregressive terms λ_2, η_2 and κ_2 are not spurious.

$$\tilde{\lambda}_t = \lambda_0 + \lambda_1 z_{t-1} + \lambda_2 \tilde{\lambda}_{t-1}, \tag{3.18}$$

$$\tilde{\eta}_t = \eta_0 + \eta_1 z_{t-1} + \eta_2 \tilde{\eta}_{t-1}, \tag{3.19}$$

$$\tilde{\kappa}_t = \kappa_0 + \kappa_1 z_{t-1} + \kappa_2 \tilde{\kappa}_{t-1}, \tag{3.20}$$

where $\tilde{\lambda}_t$ is the unrestricted skewness parameter, and $\tilde{\eta}_t$ and $\tilde{\kappa}_t$ are the unrestricted kurtosis parameters (Bali et al. (2008)). The restrictions according to the SGT definition $|\lambda_t| < 1$, $\eta_t > 2$, and $\kappa_t > 0$ are imposed through the following logistic transformation:

$$\lambda_t = -1 + 2/\left(1 + exp(-\tilde{\lambda}_t)\right), \tag{3.21}$$

$$\eta_t = 2 + exp(\tilde{\eta}_t), \tag{3.22}$$

$$\kappa_t = exp(\tilde{\kappa}_t). \tag{3.23}$$

The conditional $SGT - GARCH$ parameters are obtained from the maximization of the sample log-likelihood function

$$L = \sum_{t=1}^{T}\left[ln\left(f_Z(z_t|\lambda_t, \eta_t, \kappa_t)\right) - ln(\sigma_t)\right], \tag{3.24}$$

with respect to $\propto_0$, $\propto_1$, β_0, β_1, β_2, λ_0, λ_1, λ_2, η_0, η_1, η_2, κ_0, κ_1, κ_2 and/or γ depending on each $GARCH(1,1)$ specification and subject to positivity and stationary constraints associating with each $GARCH(1,1)$ specification.

After all conditional parameters of the return distribution are estimated, the r_t^*, which is the corresponding conditional threshold for the return r_t at a given coverage probability ϕ, is obtained directly from the solution of the following equation,

$$Pr(r_t \leq r_t^*|\Omega_{t-1}) \equiv \int_{-\infty}^{r_t^*} f_X(r_t|\Omega_{t-1})dr_t = \phi, \tag{3.25}$$

where $Pr(\cdot)$ denotes the probability, and $f_X(r_t|\Omega_{t-1})$ is the conditional probability density function of r_t.

The VaR can be obtained in terms of standardized returns as follows:

$$Pr(r_t \leq r_t^*|\Omega_{t-1}) = Pr\left(\frac{r_t-\mu_t}{\sigma_t} \leq \frac{r_t^*-\mu_t}{\sigma_t}|\Omega_{t-1}\right)$$

$$= Pr\left(z_t \leq a_t = \frac{r_t^*-\mu_t}{\sigma_t}\right)$$

$$= \int_{-\infty}^{a_t} f_Z(z_t)dz_t = \phi, \tag{3.26}$$

where μ_t and σ_t are the conditional mean and standard deviation, respectively; $f_Z(z_t)$

is the conditional SGT density function of the standardized returns; and a_t is the conditional threshold associated with the coverage probability ϕ. Given the conditional probability distribution function of standardized returns $f_Z(z_t)$, the conditional threshold a_t can be easily obtained from the solution of the equation (Bali et al. (2008))

$$\int_{-\infty}^{a_t} f_Z(z_t)dz_t = \phi. \tag{3.27}$$

In other words, the conditional threshold a_t is obtained by finding the numerical value of a_t that equalizes the area under the probability density function $f_Z(z_t)$ to the coverage probability ϕ. For example, the value of threshold a_t for $\phi = 1\%$ is constant and equal to -2.326 in case of traditional VaR analysis with standard normal distribution. However, in the more general case of conditional VaR computed from conditional SGT distribution function, the value of threshold a_t is a function of the time-varying skewness and kurtosis parameters λ_t, η_t and κ_t, not a constant. Given the value threshold a_t, in order to find the VaR r_t^*, it is simply obtained from backward transformation as follows:

$$r_t^* = \mu_t + a_t\sigma_t, \tag{3.28}$$

where μ_t is obtained from the expression (3.1) and σ_t is obtained from one of the $GARCH(1,1)$ models among the expression (3.10) to (3.17). Note that this expression (3.28) may give the conditional VaR threshold in the same horizon with the return data.

3.2 Assessment of the Performance of Conditional VaR

In order to assess the performance of estimated conditional VaR threshold both in-sample and out-of-sample, there are three tests – the unconditional coverage test, the conditional coverage test, and the dynamic quantile test; to be considered.

3.2.1 Unconditional Coverage Test

Given independence[26], Kupiec (1995) constructed the unconditional coverage test (LR^{UC}) for testing the null hypothesis that the actual and expected number of observations falling below VaR threshold (called exceedence) are statistically the same[27] (Bali et al. (2008)). The unconditional coverage test statistic is as follows:

$$LR^{UC} = 2\left[\tau ln\left(\frac{\tau}{\phi N}\right) + (N - \tau)ln\left(\frac{N-\tau}{N-\phi N}\right)\right], \qquad (3.29)$$

where N is the number of sample observations, ϕ is the coverage probability, ϕN and τ is the expected and actual number of observations falling below the VaR threshold a_t[28]. The LR^{UC} test statistic has an asymptotic Chi-square distribution with one degree of freedom, $\chi^2(1)$. The acceptance of null hypothesis refers that the computed conditional VaR threshold provides a good assessment of risk exposure. On the

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[26] It is assumed that there is no volatility clustering or volatility persistence. For example, the probability of falling below the VaR threshold of today is independent to the probability of tomorrow. The unconditional coverage test is constructed by applying binomial probability density function, which is assumed implicitly that each trial is independent. See Kupiec (1995) for further details.

[27] Under the below notation, the null hypothesis is $\tau = \phi N$.

[28] a_t is used for convenience. Since the relationship between r_t^* and a_t, and r_t and z_t are the same through equations $r_t^* = \mu_t + a_t\sigma_t$, and $r_t = \mu_t + z_t\sigma_t$, the use of a_t instead of r_t^* does not change the result from expression (4.29). N, ϕ and τ are the same regardless of using either a_t or r_t^*.

contrary, the rejection of null hypothesis indicates that the VaR threshold estimate is not accurate enough.

3.2.2 Conditional Coverage Test

The expression (3.29) is called unconditional test statistic because it simply counts exceedences (number of observations falling below VaR threshold) over the entire period. The order of the exceedence and no-exceedence in the sequence[29] does not matter, only the total number of exceedences plays a role. However, in case that the independent assumption is violated, the VaR models that ignore the presence of volatility clustering may have correct unconditional coverage, but they may have incorrect conditional coverage at any given time. In summary, the unconditional coverage test is insufficient to assess the VaR threshold when the assumption of serial independence is violated. Christoffersen (1998) developed the conditional coverage test to examine the serial independence of VaR estimates. For a given VaR estimates, the indicator variable I_t is defined as

$$I_t = \begin{cases} 1, if\ exceedence\ occurs \\ 0, if\ no\ exccedence\ occurs \end{cases} \tag{3.30}$$

The conditional coverage test statistic is constructed under null hypothesis of serial independence against the alternative of explicit first-order Markov dependence as follows:

$$LR^{IND} = 2\left[n_{00} ln\left(\frac{\Pi_{00}}{1-\Pi}\right) + n_{01} ln\left(\frac{1-\Pi_{00}}{\Pi}\right) \right.$$

$$\left. + n_{10} ln\left(\frac{\Pi_{10}}{1-\Pi}\right) + n_{11} ln\left(\frac{1-\Pi_{10}}{\Pi}\right)\right], \tag{3.31}$$

where n_{ij} is the number of observations of indicator variable I_t with value i followed by j, $\Pi_{00} = n_{00}/(n_{00} + n_{01})$, $\Pi_{10} = n_{10}/(n_{10} + n_{11})$, $\Pi = (n_{01} + n_{11})/N$, and $N = n_{00} +$

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[29] It refers to the ones and zeros in the indicator sequence introduced below.

$n_{10} + n_{01} + n_{11}$. The LR^{IND} test statistic has an asymptotic Chi-square distribution with one degree of freedom, $\chi^2(1)$. The acceptance of null hypothesis indicates that the serial independence assumption is held and it suffices to use the unconditional coverage test to assess the VaR threshold. On the other hand, the rejection of null hypothesis refers that the unconditional coverage test is of limited use since it will clarify inaccurate VaR threshold estimates as "acceptably accurate" (Bali et al. (2008)).

3.2.3 Dynamic Quantile Test

Although the conditional coverage test of Christoffersen (1998) can detect the presence of serial dependence in the indicator sequence $\{I_t\}$, Engle and Manganelli (2004) argued that this is only a necessary, but not sufficient condition to assess the performance of VaR from some estimation procedures, i.e. a quantile model. The dynamic quantile test proposed by Engle and Manganelli (2004) provided another tool for investigating the serial independence of VaR estimates like the conditional coverage test of Christoffersen (1998) but it is more flexible for its application to any VaR estimates regardless of the estimation procedure. The dynamic quantile test statistic[30] is as follows:

$$DQ = \frac{\hat{B}'(X'X)\hat{B}}{\omega(1-\omega)},$$
(3.32)

where $\hat{B}$ is the OLS estimates of B from the artificial regression $Hit_t = XB + \epsilon_t$, Hit_t is a sequence of indicator variable defined as $Hit_t = I(r_t < -VaR_t(\omega)) - \omega$, $I(\cdot)$ is an indicator function, ω is given confidence level[31], $VaR_t(\cdot)$ is conditional VaR threshold at given confidence level, X is a $N \times k$ matrix whose first column is a column of ones

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[30] This is the DQ test statistic for out-of-sample case. It is simpler and more flexible for applying to any estimation procedure rather than the in-sample DQ test statistic.

[31] Confidence level is equal to one minus coverage probability or $\omega = 1 - \phi$.

and the remaining column are additional explanatory variables including five lags of Hit_t and the current VaR threshold[32] (Bali et al. (2008)). The DQ test statistic has an asymptotic Chi-square distribution with seven degree of freedom, $\chi^2(7)$. Its implication is similar to of the conditional coverage test of Christoffersen (1998). The acceptance of null hypothesis guarantees the serial independence of the VaR estimates and therefore, supports the sufficiency of applying the unconditional coverage test of Kupiec (1995).

3.3 Data

This research uses daily returns on the SET value-weighted index[33] from January 1976 to December 2010 (8,605 observations). This index includes all stock trading on the Stock Exchange of Thailand (SET) and gives relatively more weight on the high-price stocks[34]. The daily realized index return is calculated by the following equation as:

$$r_t = 100\big(ln(p_t) - ln(p_{t-1})\big), \tag{3.33}$$

where p_t is the stock price index level for period t.

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[32] It is suggested by Berkowitz et al. (2005).

[33] See SET (2010) for further details.

[34] See Standard & Poor's (2009) for further details.

3.4 Research Design

The procedure used in this research can be summarized as follows:

(1) Data Preparation: By using the expression (3.33), daily returns will be obtained from the SET value-weighted index between January 1976 and December 2010. The model's performance will be assessed in two parts, in-sample and out-of-sample analyses. The return series will be chosen accordingly between estimation and prediction sample as follows:

Table 3.1: Estimation and Prediction Sample for In-sample and Out-of-Sample Analyses

	Estimation Sample	Prediction Sample
In-sample analysis	1976-2010	1976-2010
Out-of-sample analysis	1998-2009	2010

(2) Estimation of Parameters: The empirical returns will be modeled by the expression (4.1). Mean μ_t, and standard deviation σ_t are assumed to follow $AR(1)$, and 8 types of $GARCH(1,1)$ processes, respectively. Standardized return z_t is assumed to be the SGT density function with additional higher-order parameters, controlled skewness and kurtosis. The dynamics in λ_t, η_t, and κ_t are modeled according to the expression (3.18), (3.19), and (3.20). Notably, the boundary of higher-order parameters will be met by the expression (3.21), (3.22) and (3.23). Using estimation sample, the estimated parameters will be obtained by the maximization of log-likelihood function in the expression (3.25) (Maximum Likelihood Estimation). For example, in case of $EGARCH$ model, we will obtain $\hat{\alpha}_0$, $\hat{\alpha}_1$, $\hat{\beta}_0$, $\hat{\beta}_1$, $\hat{\beta}_2$, $\hat{\gamma}$, $\hat{\lambda}_0$, $\hat{\lambda}_1$, $\hat{\lambda}_2$, $\hat{\eta}_0$, $\hat{\eta}_1$, $\hat{\eta}_2$, $\hat{\kappa}_0$, $\hat{\kappa}_1$, $\hat{\kappa}_2$.

(3) Calculation of VaR: Using all estimates with the current realized return r_t, the conditional VaR r_t^* will be obtained by solving the integral equation (3.26). Note that this task can be done by solving the expression (3.27) and (3.28) also.

(4) Assessment of performance of conditional VaR: The realized return r_t and conditional VaR r_t^* for $t = 1, ..., T$ will be used together to assess whether the actual and expected number of observations falling below VaR threshold are statistically the same by the unconditional coverage test in expression (3.29). Furthermore, the conditional coverage and dynamic quantile tests in expression (3.31) and (3.32), respectively, are also used to verify the underlying assumption of the unconditional coverage test. It will guarantee and support the sufficiency of applying the unconditional coverage test.

Chapter 4 Empirical Results

4.1 Descriptive Statistics

Table 4.1 provides basic descriptive statistics on the data between January 1976 and December 2010 (8,605 observations). In the second column, the unconditional mean of daily realized return on SET index computed according to the expression (3.33) is 0.0293 with the standard deviation of 1.4967. The maximum and minimum values are 11.3495 and -16.0633, respectively. The skewness statistic is negative and significant at 1% level. It implies that the distribution of daily realized return on SET index is skewed to the left. The excess kurtosis is more than zero and significant at 1% level, implying that the distribution of realized returns has thicker tails than the normal distribution. Accordingly, the Jarque-Bera statistic is very large and significant. It rejects the normality assumption.

The last column in Table 4.1 presents the descriptive statistics on the standardized residual, which is obtained from a maximum likelihood estimation of a following *GARCH* model:

$$r_t = \alpha_0 + \alpha_1 r_{t-1} + \sigma_t z_t,$$

$$\sigma_t^2 = \beta_0 + \beta_1 \sigma_{t-1}^2 z_{t-1}^2 + \beta_1 \sigma_{t-1}^2,$$

where z_t is the standardized residual for period t and assumed to have a standard normal distribution. The conditional mean of the standardized residual is -0.0003 with a standard deviation of 1.0408. The maximum and minimum values are 5.8729 and   -22.5196, respectively. The skewness statistic is significantly negative at 1% level, implying that the distribution of the standardized residual is skewed to the left. The excess kurtosis statistic on the standardized residual is much larger than

that on the raw returns and significant at 1% level, implying that the distribution of the standardized residual has much thicker tails than the standard normal distribution. The Jarque-Bera statistic is still very large, rejecting the assumption of normality.

Table 4.1: Descriptive Statistics

Statistics	Realized Index Return	Standardized Residual
Mean	0.0293	-0.0003
Maximum	11.3495	5.8729
Minimum	-16.0633	-22.5196
Std. Dev.	1.4967	1.0408
Skewness	-0.1013	-1.2214
	$(0.0264)^{**}$	$(0.0264)^{**}$
Excess Kurtosis	8.6761	27.3529
	$(0.0528)^{**}$	$(0.0528)^{**}$
Jarque-Bera	26997.5663^{**}	270330.4425^{**}

Note: The table presents the descriptive statistics of the realized returns and the standardized residuals on the daily SET value-weighted price index from January 1976 to December 2010 (8,605 observations). Standard errors of the skewness and excess kurtosis given in the parentheses are calculated as $\sqrt{6/n}$ and $\sqrt{24/n}$, respectively. Jarque-Bera, $JB = n[(S^2/6) + (K^2/24)]$, is the formal test statistic for testing whether the underlying returns are normally distributed, where n, S, and K denote the number of observations, skewness, and excess kurtosis, respectively. The JB statistic is distributed as the Chi-square with two degrees of freedom. *, ** denote significance at the 5% and 1% level, respectively.

4.2 Estimates of Alternative Unconditional Distributions

Table 4.2 presents the constant parameter estimates among alternative unconditional distribution functions: the SGT, the generalized t, the skewed t, the symmetric t, and the normal distribution functions for entire sample (January 1976 to December 2010). Since the latter four distributions can be considered as special cases of the SGT distributions, they can be shown in terms of the SGT distribution while some parameters are constant.

The tail-thickness parameter η of the symmetric t distribution is about 2.1533 and significant at 1% level. The likelihood ration (LR) test statistic from testing the null hypothesis that $\eta = \infty$ can be calculated from the likelihood obtained from the symmetric t and the normal distribution models as: $LR = (-2)[(-5677.60) - (-4331.97)] = 2691.26$. The LR statistic is distributed as the Chi-square with one degree of freedom. As a result, the null hypothesis is strongly rejected, implying that the empirical distribution of daily SET returns does not follow a normal distribution. The peakedness parameter κ of the generalized t distribution is about 1.0525 and significant at 1% level. The LR statistic of the null hypothesis that $\kappa = 2$ can be calculated likewise from the likelihood obtained from the generalized t and the symmetric t distribution models as: $LR = (-2)[(-4331.97) - (-4254.98)] = 153.98$. Therefore, it strongly rejects the null hypothesis, implying that the empirical return distribution is peaked around the mean and does not follow the symmetric t distribution. The skewness parameter λ of the skewed t distribution is about 0.0065 but not significant even at 60% level. The LR statistic from null hypothesis that $\lambda = 0$ is calculated from the skewed t and symmetric t distribution models as: $LR = (-2)[(-4331.97) - (-4331.84)] = 0.26$. It implies that the empirical distribution of daily realized returns is symmetric.

The LR statistic in the last column is used to test null hypothesis of alternative distribution against the SGT distribution. The result indicates strong rejection of the skewed t, the symmetric t, and the normal distribution. However, it cannot reject the null hypothesis in case of the generalized t distribution. With the insignificance of skewness parameter λ, it seems that the generalized t distribution will be the most appropriated since it has thicker tails than the normal, peaked around the mean and symmetric.

Nevertheless, it might not be the case. By law of iterated expectation[35], zero unconditional expectation does not imply that conditional expectation must be zero. Since we are dealing with conditional model of returns, if the generalized t density is applied as conditional distribution of standardized returns, it refers that the skewness parameter λ is assumed to be zero and the conditional distribution of returns is symmetric for all t. This may not be true, however. The estimates of unconditional distribution from the *SGT* density are quite supportive to such possibility. All parameters including the skewness parameter λ are highly significant at 1% level, implying that the skewness parameter has some effects to the shape of return distribution. Without strong rejection of the *SGT* distribution assumption, the *SGT* will be more appropriate comparing to the generalized t. The zero skewness parameter can be obtained if we start from the *SGT*. On the other hand, the dynamics in skewness parameter will not be able to observe if we start from the generalized t.

Table 4.2: Maximum Likelihood Estimates of Alternative Unconditional Distributions

Distribution	μ	σ	λ	η	κ	$\mathcal{L}$	LR
SGT	0.0347 (3.99)**	1.5001 (54.23)**	0.0163 (2.71)**	6.4878 (5.84)**	1.0466 (21.01)**	-4254.05	-
Generalized t	0.0167 (1.41)	1.5000 (52.48)**	0	6.3928 (5.52)**	1.0525 (19.7)**	-4254.98	1.86
Skewed t	0.0376 (2.29)*	2.1568 (10.75)**	0.0065 (0.51)	2.3245 (29.75)**	2	-4331.84	155.58
Symmetric t	0.0313 (2.88)**	2.1533 (10.8)**	0	2.3259 (29.76)**	2	-4331.97	155.85
Normal	0.0292 (1.81)	1.4966 (131.18)**	0	∞	2	-5677.60	2847.09

Note: This table presents the parameter estimates of various alternative unconditional distribution in terms of the *SGT* since it nests the generalized t, the skewed t, the symmetric t and the normal distributions. Numbers in

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[35] See theorem 4.4.3 in Casella and Berger (2002) for further details.

parenthesis are t statistic for hypothesis that the estimates are statistically significant. [*], [**] denote significance at the 5% and 1% level, respectively.

4.3 Estimates of GARCH Model Based on the SGT with Constant Skewness and Kurtosis Parameters

Table 4.3 presents the maximum likelihood estimates for various $GARCH(1,1)$-type models based on the SGT density with constant skewness and kurtosis parameters, using entire sample (January 1976 to December 2010). The constant term in the conditional mean equation α_0 is significant in the $GARCH$, $IGARCH$, $GJRGARCH$, $QGARCH$ and $APGARCH$ but not in the $EGARCH$, $TGARCH$ and $TSGARCH$ models. However, the $AR(1)$ coefficient α_1 in all models is highly significant at 1% level in the range of 0.15 to 0.17. These results indicate first-order autocorrelation in the daily returns on SET index.

The $GARCH$ parameters in the conditional variance equation quite vary depending on the models. For parameters in the symmetric-$GARCH$-type models, the $GARCH$, $IGARCH$, and $TS-GARCH$, they are all highly significant at 1% level. The estimated results reveal the presence of significant volatility persistence of the SET index returns. For example, the sum of parameter β_1 and β_2 is close to one, implying the existence of strongly persistent volatility in the returns. For the asymmetric-$GARCH$-type models, the $EGARCH$, $GJR-GARCH$, $QGARCH$, $TGARCH$, and $APGARCH$, while all parameter β_0, β_1 and β_2 are statistically significant at 1% level except the β_0 of $EGARCH$ model, the asymmetry coefficient γ in all models is insignificant, implying that there is no leverage effect in the returns on the SET index.

The estimated values of constant kurtosis parameters η and κ are highly significant at 1% level in all models. In particular, η s are significantly greater than 3 and κ s are less than 2, indicating that the standardized returns are peaked around

the mean and have fatter tails. However, it is inconclusive for the estimate of skewness parameter λ. Specifically, half of them are statistically different from zero at 5% level (the $GARCH$, $IGARCH$, $GJR-GARCH$, and $APGARCH$) but another half are not (the $EGARCH$, $QGARCH$, $TGARCH$, and $TS-GARCH$). These results are quite harmonious with the estimated results of the unconditional distribution in the previous section that there is no strong evidence of the zero unconditional skewness parameter λ.

Table 4.3: Maximum Likelihood Estimates of the SGT-GARCH Models with Constant Skewness and Kurtosis Parameters

	GARCH	IGARCH	EGARCH	GJRGRH	QGARCH	TGARCH	TSGRCH	APGRCH
α_0	0.0201 (2.54)*	0.0214 (2.69)**	0.0142 (1.56)	0.0201 (2.55)*	0.0215 (2.70)**	0.0097 (1.26)	0.0119 (1.50)	0.0201 (2.55)*
α_1	0.1608 (14.21)**	0.1583 (13.97)**	0.1599 (11.51)**	0.1609 (14.22)**	0.1610 (14.23)**	0.1560 (14.29)**	0.1562 (15.29)**	0.1609 (14.22)**
β_0	0.0082 (5.63)**	0.0095 (6.37)**	0.0078 (1.17)	0.0085 (5.64)**	0.0082 (5.70)**	0.0122 (5.63)**	0.0118 (5.58)**	0.0085 (5.64)**
β_1	0.1419 (16.17)**	-	0.3094 (17.35)**	0.1352 (15.2)**	0.1408 (16.05)**	0.1631 (15.2)**	0.1677 (15.8)**	0.1439 (16.18)**
β_2	0.8555 (97.38)**	0.8508 (97.60)**	0.9824 (373.85)**	0.8534 (95.9)**	0.8565 (97.55)**	0.8658 (105.21)**	0.8674 (106.77)**	0.8534 (95.9)**
γ	-	-	-0.0068 (-0.89)	0.0178 (1.61)	0.0102 (1.54)	-0.0126 (-1.42)	-	0.0310 (1.63)
λ	0.0272 (2.00)*	0.0269 (1.96)*	0.0115 (0.85)	0.0317 (2.29)*	0.0250 (1.82)	-0.0018 (-0.18)	-0.0028 (-0.28)	0.0317 (2.29)*
η	7.4477 (9.06)**	7.3064 (8.98)**	6.2741 (8.43)**	7.4661 (9.07)**	7.4106 (9.07)**	6.2326 (8.48)**	6.2089 (8.5)**	7.4661 (9.07)**
κ	1.8240 (22.38)**	1.8208 (22.11)**	1.8523 (20.1)**	1.8243 (22.42)**	1.8265 (22.3)**	1.8464 (20.16)**	1.8481 (20.13)**	1.8243 (22.42)**
$\mathcal{L}$	-12825.02	-12819.13	-12787.52	-12823.69	-12823.85	-12791.80	-12792.82	-12823.69

Note: Numbers in parenthesis are t statistic for hypothesis that the estimates are statistically significant. *, ** denote significance at the 5% and 1% level, respectively.

4.4 Estimates of GARCH Model Based on the SGT with Time-varying Skewness and Kurtosis Parameters

Table 4.4 presents the parameter estimates of alternative $GARCH(1,1)$-type models based on SGT distribution with time-varying skewness and kurtosis using the data between January 1976 and December 2010 (in-sample estimation). The significance of constant term α_0 in mean equation changes in the $EGARCH$, $GJR - GARCH$, and $TS - GARCH$ models. With time-varying skewness and kurtosis, all parameters β_0, β_1 and β_2 in variance equation are statistically significant. The asymmetry parameter γ, which is insignificant in all asymmetric-$GARCH$-type models with constant skewness and kurtosis, now turns to be significant in the $GJR - GARCH$, $TGARCH$, and $APGARCH$ models. It refers that there exists an asymmetric volatility response to past positive and negative information shocks in the SET index returns.

The parameter η_1 and κ_1 in the $EGARCH$ and $GJR - GARCH$ models is dropped in order to avoid the spurious correlation problem in the intermediate step[36]. Out of the $EGARCH$ and $GJR - GARCH$ model for η_1 and κ_1, parameter λ_1, η_1 and κ_1 are all statistically significant, implying that the dynamics of conditional skewness and kurtosis depend on past information z_{t-1}. However, parameter λ_2, η_2 and κ_2 are insignificant in almost all models (except λ_2 in the $GARCH$, $QGARCH$, and $APGARCH$ models). It refers that the conditional skewness and kurtosis are not determined by their lagged term. The constant coefficient λ_0, η_0 and κ_0 in the conditional skewness and kurtosis equation quite vary upon $GARCH$ specifications. Note that the constant term λ_0 in the conditional skewness is statistically different from zero in the $GARCH$, $IGARCH$, $QGARCH$, and $APGARCH$ models but not for the others. The results are quite consistent with the previous section that the unconditional skewness may not be

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[36] See footnote 25 in chapter 3 for further details.

zero. Together with the significance of λ_1, it also supports the use of the *SGT* distribution instead of the generalized t distribution mentioned in section 4.2. If conditional skewness was assumed to be zero for all period at first place, the effect of past information z_t will be overlooked in the conditional model.

LR in the last row is the likelihood ratio test statistic against the null hypothesis of constant skewness and kurtosis with the *SGT* distribution. They are all highly significant at 1%, implying that the conditional $SGT - GARCH$ models with time-varying skewness and kurtosis provide a better fit for the SET index returns than the $SGT - GARCH$ models with constant skewness and kurtosis.

Table 4.4: Maximum Likelihood Estimates of the SGT-GARCH Models with

Time-Varying Skewness and Kurtosis Parameters

	GARCH	IGARCH	EGARCH	GJRGRH	QGARCH	TGARCH	TSGRCH	APGRCH
α_0	0.0199 (2.58)**	0.0212 (2.73)**	0.0142 (2.00)*	0.0138 (1.78)	0.0213 (2.73)**	0.0132 (1.81)	0.0169 (2.20)*	0.0184 (2.38)*
α_1	0.1692 (15.25)**	0.1684 (15.09)**	0.1773 (14.31)**	0.1745 (15.33)**	0.1692 (15.23)**	0.1685 (19.44)**	0.1664 (14.94)**	0.1698 (15.32)**
β_0	0.0084 (5.60)**	0.0098 (6.37)**	0.0068 (2.09)*	0.0074 (4.87)**	0.0084 (5.65)**	0.0122 (5.62)**	0.0120 (5.74)**	0.0089 (5.64)**
β_1	0.1471 (16.30)**	-	0.3210 (17.26)**	0.1524 (15.36)**	0.1462 (16.18)**	0.1618 (15.99)**	0.1699 (16.52)**	0.1501 (16.39)**
β_2	0.8501 (94.15)**	0.8451 (94.56)**	0.9812 (354.6)**	0.8333 (84.98)**	0.8510 (94.16)**	0.8622 (105.9)**	0.8649 (105.84)**	0.8472 (92.53)**
γ	-	-	-0.0130 (-1.66)	0.0746 (6.44)**	0.0081 (1.16)	-0.0253 (-3.25)**	-	0.0453 (2.24)*
λ_0	0.0436 (2.04)*	0.0449 (2.03)*	0.0371 (1.50)	0.0432 (1.87)	0.0428 (2.00)*	0.0437 (1.77)	0.0438 (1.78)	0.0445 (2.08)*
λ_1	0.1285 (5.41)**	0.1323 (5.43)**	0.1829 (7.22)**	0.1210 (5.74)**	0.1264 (5.32)**	0.1273 (5.01)**	0.1194 (4.80)**	0.1344 (5.58)**
λ_2	0.2626 (1.98)*	0.2444 (1.80)	0.1416 (1.17)	0.2514 (1.69)	0.2632 (1.96)*	0.1984 (1.18)	0.2024 (1.14)	0.2617 (2.07)*
η_0	2.6171 (3.90)**	2.6367 (4.13)**	1.5864 (0.03)	1.9671 (0.04)	2.5956 (3.88)**	2.1769 (3.66)**	2.2165 (3.84)**	2.6703 (3.89)**
η_1	0.0876 (2.15)*	0.0884 (2.09)*	-	-	0.0855 (2.13)*	0.1084 (2.52)*	0.1049 (2.53)*	0.0900 (2.11)*
η_2	-0.4675 (-1.35)	-0.4954 (-1.51)	-0.0090 (0.00)	-0.2660 (-0.01)	-0.4611 (-1.33)	-0.4174 (-1.22)	-0.4209 (-1.28)	-0.4876 (-1.39)
κ_0	0.6006 (1.96)*	0.5749 (2.13)*	0.5878 (12.26)**	-2.3525 (-2094.84)**	0.5834 (1.98)*	-2.4674 (-2.14)*	-2.2658 (-2.17)*	-2.6171 (-1.75)
κ_1	-0.0425 (-2.11)*	-0.0425 (-2.09)*	-	-	-0.0433 (-2.15)*	-0.0517 (-2.02)*	-0.0567 (-2.24)*	-0.0424 (-1.93)
κ_2	-0.0361 (-0.07)	0.0022 (0.00)	0.0015 (0.04)	-0.0187 (-0.83)	-0.0044 (-0.01)	-0.0687 (-0.14)	0.0199 (0.04)	-0.1242 (-0.19)
$\mathcal{L}$	-12804.74	-12798.99	-12760.92	-12786.07	-12804.07	-12772.84	-12776.00	-12802.24
LR	40.55**	40.28**	53.21**	75.24**	39.55**	37.93**	33.65**	42.90**

Note: Numbers in parenthesis are t statistic for hypothesis that the estimates are statistically significant. *, **

denote significance at the 5% and 1% level, respectively.

4.5 Assessment of In-sample VaR Performance

Table 4.5 presents statistics on the *VaR* threshold of all models for the coverage probabilities ϕ of 1%, 1.5%, 2%, 2.5%, and 5% using the sample between January 1976 and December 2010 (in-sample analysis). The first row for each coverage probability presents the actual and expected (*Actl/Expt*) number of returns that fall below each threshold. The second row presents the unconditional coverage tests (LR^{uc}) for testing the null hypothesis that the actual and the expected number of observations falling below each threshold are statistically the same[37]. The last row presents the conditional coverage tests (LR^{ind}) for testing whether the unconditional coverage tests are reliable[38].

The conditional coverage test statistics LR^{ind} in all models and coverage probabilities cannot reject the null hypothesis of the serial independent assumption of the unconditional coverage test, indicating that the assessment of *VaR* threshold can rely on the unconditional coverage test statistics LR^{uc}.

The unconditional coverage test statistics shows that the *APGARCH* model is the most inaccurate for predicting the *VaR* threshold since they rejects the null hypothesis at all coverage probability level. The *GARCH*, *IGARCH*, and *QGARCH* models are all accurate only at high coverage probability but not the low one (except the *IGARCH* model at 2%). In contrast, the *EGARCH*, *GJR* − *GARCH*, and *TGARCH* model do poorly for the high coverage probabilities but become better when it goes further to the tail of the return distribution (low coverage probabilities).

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[37] See section 3.2.1, chapter 3.

[38] See section 3.2.2, chapter 3.

The $TS - GARCH$ model provides the best assessment of the risk exposure of a portfolio mimicking the SET index return since the null hypothesis cannot be rejected at all coverage probability level. It implies that the VaR threshold obtained from the $TS - GARCH$ model based on SGT distribution with time-varying skewness and kurtosis is accurate and consistent regardless of coverage probability chosen.

Table 4.5: In-sample VaR Performance of the SGT-GARCH Models with Time-varying Skewness and Kurtosis parameters

	GARCH	IGARCH	EGARCH	GJRGRH	QGARCH	TGARCH	TSGRCH	APGRCH
1.0%								
$Actl/Expt$	125/86	116/86	74/86	75/86	121/86	78/86	92/86	330/86
LR^{uc}	15.64	9.50	1.78**	1.49**	12.75	0.78**	0.41**	406.41
LR^{ind}	0.02**	0.24**	0.18**	1.32**	0.35**	0.11**	0.00**	0.88**
1.5%								
$Actl/Expt$	175/129	169/129	128/129	126/129	172/129	107/129	127/129	359/129
LR^{uc}	14.96	11.45	0.01**	0.07**	13.15	4.06*	0.03**	281.02
LR^{ind}	0.05**	0.14**	0.53**	0.01**	0.09**	0.09**	0.01**	1.09**
2.0%								
$Actl/Expt$	219/172	206/172	200/172	190/172	218/172	134/172	162/172	381/172
LR^{uc}	12.04	6.43*	4.40*	1.85**	11.55	9.29	0.61**	193.09
LR^{ind}	0.06**	0.00**	0.03**	0.01**	0.05**	0.00**	0.43**	1.57**
2.5%								
$Actl/Expt$	273/215	256/215	280/215	269/215	272/215	177/215	204/215	402/215
LR^{uc}	14.76	7.54	18.38	12.86	14.28	7.35	0.59**	133.22
LR^{ind}	0.62**	0.05**	0.00**	0.04**	0.23**	0.13**	0.27**	0.98**
5.0%								
$Actl/Expt$	475/430	450/430	617/430	620/430	474/430	385/430	391/430	523/430
LR^{uc}	4.77*	0.95**	75.75	78.07	4.56*	5.16*	3.86*	19.79
LR^{ind}	0.00**	0.09**	0.14**	1.22**	0.00**	0.85**	1.56**	0.29**

Note: *, ** denote significance at the 5% and 1% level, respectively.

4.6 Assessment of Out-of-sample VaR Performance

Table 4.6 presents statistics on the *VaR* threshold of all models for the coverage probabilities ϕ of 1%, 1.5%, 2%, 2.5%, and 5% using the sample between January 2000 and December 2009 for estimation, and the last quarter of December 2010 for prediction (out-of-sample analysis). In addition to the conditional coverage statistic, the last row of each coverage probabilities presents the dynamic quantile test (*DQ*) for testing whether it suffices to use the unconditional coverage test to assess the *VaR* threshold performance[39].

The results from the conditional coverage and dynamic quantile statistics show that all unconditional coverage statistics are reliable and suffice to assess the performance of *VaR* threshold. The unconditional coverage test statistics in all models strongly indicate that all models provide accurate *VaR* threshold in case of out-of-sample analysis.

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[39] See section 3.2.3, chapter 3.

Table 4.6: Out-of-Sample VaR Performance of the SGT-GARCH Models with

Time-varying Skewness and Kurtosis parameters

	GARCH	IGARCH	EGARCH	GJRGRH	QGARCH	TGARCH	TSGRCH	APGRCH
1.0%								
Actl/Expt	1/0.62	2/0.62	1/0.62	1/0.62	1/0.62	0/0.62	1/0.62	1/0.62
LR^{uc}	0.19**	1.96**	0.19**	0.19**	0.19**	1.25**	0.19**	0.19**
LR^{ind}	0.03**	0.14**	0.03**	0.03**	0.03**	NA	0.03**	0.03**
DQ	0.56**	1.81**	1.25**	0.69**	0.59**	NA	0.62**	2.01**
1.5%								
Actl/Expt	1/0.93	2/0.93	1/0.93	1/0.93	1/0.93	1/0.93	1/0.93	1/0.93
LR^{uc}	0.01**	0.94**	0.01**	0.01**	0.01**	0.01**	0.01**	0.01**
LR^{ind}	0.03**	0.14**	0.03**	0.03**	0.03**	0.03**	0.03**	0.03**
DQ	0.17**	0.98**	0.64**	0.28**	0.18**	0.33**	0.25**	1.15**
2.0%								
Actl/Expt	1/1.24	2/1.24	1/1.24	1/1.24	1/1.24	1/1.24	1/1.24	1/1.24
LR^{uc}	0.05**	0.40**	0.05**	0.05**	0.05**	0.05**	0.05**	0.05**
LR^{ind}	0.03**	0.14**	0.03**	0.03**	0.03**	0.03**	0.03**	0.03**
DQ	0.12**	0.73**	0.48**	0.22**	0.13**	0.26**	0.19**	0.86**
2.5%								
Actl/Expt	1/1.55	2/1.55	1/1.55	1/1.55	1/1.55	1/1.55	1/1.55	1/1.55
LR^{uc}	0.22**	0.12**	0.22**	0.22**	0.22**	0.22**	0.22**	0.22**
LR^{ind}	0.03**	0.14**	0.03**	0.03**	0.03**	0.03**	0.03**	0.03**
DQ	0.22**	0.69**	0.49**	0.29**	0.22**	0.32**	0.27**	0.81**
5.0%								
Actl/Expt	2/3.1	4/3.1	1/3.1	1/3.1	2/3.1	2/3.1	1/3.1	2/3.1
LR^{uc}	0.47**	0.25**	2.01**	2.01**	0.47**	0.47**	2.01**	0.47**
LR^{ind}	0.14**	0.56**	0.03**	0.03**	0.14**	0.14**	0.03**	0.14**
DQ	0.86**	2.92**	1.45**	1.36**	1.04**	1.37**	1.32**	1.36**

Note: *, ** denote significance at the 5% and 1% level, respectively.

Chapter 5 Conclusions

5.1 Conclusions

With complexity in the current financial market, Value-at-Risk (VaR) is one of primary tool used to assess the risk in financial market. Despite the simplicity of its concept, an accurate calculation of conditional VaR is still statistically challenging. This research proposes an alternative to compute parametric VaR called $SGT - GARCH - VaR$ approach. The traditional distributional assumption of Gaussian density has been investigated among other more flexible distribution, for example, the symmetric t, skewed t, generalized t, and skewed generalized t (SGT) distribution. The conditional volatility is assumed to follow 8 types of $GARCH(1,1)$ process including symmetric and asymmetric ones. Furthermore, the conventional assumption in conditional VaR calculation that distribution of standardized returns is iid is also relaxed. We allow higher-order moments of the SGT distribution to rely on the past information set similar to $ARCH$ process by defining the skewness, tail-thickness, and peakedness parameters of the SGT density as an auto regressive process.

The maximum likelihood estimates of the $SGT - GARCH$ models with time-varying skewness and kurtosis show that the conditional volatility is in favor of symmetric-$GARCH$-type models (the $GARCH$, $IGARCH$, $TS - GARCH$). The time-varying conditional skewness, tail-thickness, and peakedness of the SGT distribution mostly depend on the constant term and past information set, not their autoregressive term. The likelihood ratio test statistics against the null hypothesis of $SGT - GARCH$ models with constant skewness and kurtosis indicate that the conditional $SGT - GARCH$

models with time-varying skewness and kurtosis provide a better fit for the SET index returns.

Then, we investigate the role of conditional skewness and kurtosis in the estimation of the conditional VaR by using the unconditional coverage test, the conditional coverage test, and the dynamic quantile test to evaluate the performance of the conditional $SGT-GARCH-VaR$ approach. The in-sample performance results indicate that the conditional $SGT-VaR$ approach with time-varying skewness and kurtosis in case of the $TS-GARCH$ provides very good predictions of market risks regardless of coverage probability chosen. However, the performance results for out-of-sample analysis are still unclear. The $SGT-VaR$ approach with conditional skewness and kurtosis in all $GARCH$-type can provide accurate VaR threshold. There is no superior $GARCH$ specification among others.

5.2 Policy Implications

The policy implications can be considered in two respects. Firstly, since the risk-based capital charges in the Basel II Accord rely on the calculation of credit and market risks by VaR method, the accurate calculation of VaR threshold in this research will be beneficial indirectly to the stability and soundness of Thailand's financial system as the financial institutions will have adequate buffer against the shocks during financial crisis. Secondly, the $SGT-GARCH-VaR$ proposed in this research can be applied for any private sector as an alternative tool for risk management.

5.3 Limitations of the Study

The daily return on the SET value-weighted index used in this research is just only a case study for investigating the performance of the $SGT - GARCH - VaR$ approach. In order to bring it into practice, more empirical research is necessary to be explored in the future to guarantee an accuracy of this new approach. Furthermore, there is discrepancy between the in-sample and out-of-sample analysis. It may be caused from small prediction sample. All of these issues could be considered as a further step in the research.

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